Math 213, Spring 2015, Exam 1

1. Consider the vectors $\boldsymbol{u} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{v} = 2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, and $\boldsymbol{w} = -\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$.
   a. Find the area of the parallelogram determined by $\boldsymbol{u}$ and $\boldsymbol{v}$.
   b. Find the volume of the parallelepiped determined $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{w}$.
   c. Find the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$.

2. Find an equation for the plane that passes through the point $(1,2,3)$ parallel to $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{v} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$.

3. The line L: $x = 3 + 2t$, $y = 2t$, $z = t$ intersects the plane $x + 3y - z = -4$ in a point P. Find the coordinates of P and find equations of the line in the plane through P and perpendicular to L.

4. Find the distance between the line L_1 through the points $A(1,0,-1)$ and $B(-1,1,0)$ and the line L_2 through the points $C(3,1,-1)$ and $D(4,5,-2)$.

5. Sketch the graph of $\boldsymbol{r}(t) = \sin^2 t\,\boldsymbol{i} + \cos^2 t\,\boldsymbol{j} - \boldsymbol{k}$, $0 \le t \le \frac{\pi}{2}$, and show that it lies in the plane $3x + 3y - 5z = 8$.

6. Consider the vector-valued function $\boldsymbol{r}(t) = \sin t\,\boldsymbol{i} + 5\cos t\,\boldsymbol{j}$, $t \ge 0$.
   a. Sketch the image of $\boldsymbol{r}(t)$.
   b. Sketch $\boldsymbol{r}\left(\frac{\pi}{4}\right)$ and $\boldsymbol{r}'\left(\frac{\pi}{4}\right)$ on the image of $\boldsymbol{r}(t)$.
   c. Find the equations of the line tangent to the curve $\boldsymbol{r}(t)$ at $t = \frac{\pi}{4}$.
   d. Suppose that a particle follows the path $\boldsymbol{r}(t)$ until it flies off on a tangent at $t = \frac{\pi}{4}$. Find the coordinates of the particle at $t = \frac{3\pi}{4}$.

7.
   a. Identify and sketch the surface $x^2 + y^2 + z^2 - 8x + 6z = 0$.
   b. Sketch the circular paraboloid $x = y^2 + z^2$. Give the parametric equations for the curve of intersection of this paraboloid with the plane $x = 1$.